Proposed solutions for tutorial 7

*Intermediate Microeconomics (UTS 23567)**

Preliminary and incomplete

Available at https://backwardinduction.blog/tutoring/

Office hours on Mondays from 9 am till 10 am in building 8 on level 9

Please whatsapp me on 0457871540 so I could meet you at the door, I don't have an internal phone

Also please whatsapp if you have questions, I won't be able to answer through whatsapp but I will give answer during office hours or, since you are very unlikely to have ask a unique question, in the beginning of next tutorial

Sergey V. Alexeev

8 of May, 2018

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*Questions for the tutorial were provided by Massimo Scotti, slide and textbook are by Nechyba (2016). Solutions and commentary are by Sergey Alexeev (e-mail: sergei.v.alexeev@gmail.com). (Btw this is a much better book Varian (1987))

Question 1

Suppose technology is given by

$$f(\ell, k) = k\ell$$

In Tutorial 6 you saw that, in this case, the least-cost input combination to produce

$$x = 8$$

when

$$w = \$1$$

and

$$r = \$2$$

is

$$(\ell, k) = (4, 2)$$

1.a.
What is the cost of producing 8 units of output when the price of labour is \$1 and the price of capital \$2?

Answer to 1.a

Recall that

$$(4, 2) = (\ell^*, k^*) = \arg\min_{\ell,k} \begin{cases} w\ell + rk \\ \text{s.t. } f(\ell, k) = 8 \end{cases}$$

i.e. being optimal the input already contains information on an isoquant

Thus

$$C = w\ell^* + rk^* = 1 \times 4 + 2 \times 2 = 8$$

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1.b
In the graph above, show what happens when r decreases to \$1

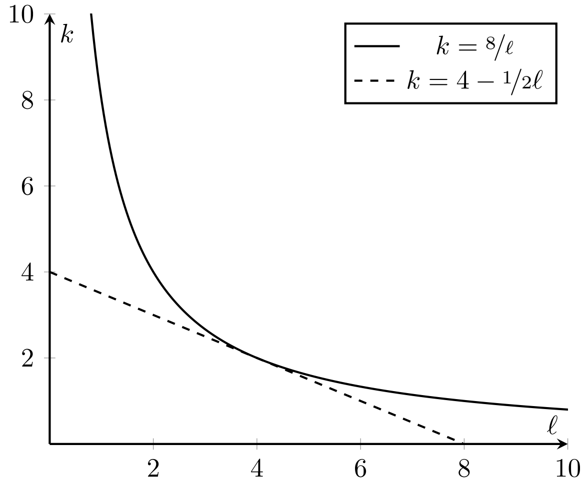
Answer to 1.b

Intuitively a decrease in r should lead to increase of the usage of k in production.[1]

We can calculate costs

$$\begin{aligned} w\ell + rk &= 8 \\ k &= \frac{8}{r} - \frac{w}{r}\ell \\ k &= 4 - \frac{1}{2}\ell \end{aligned}$$

and plot it with an isoquant



[1]This is what your maths has to reflect. Always try to envision the answer with your minds before doing mathematics, it'll save you time in the long run. Mathematics is an intuition expressed formally, thus mathematics without intuition is not mathematics.

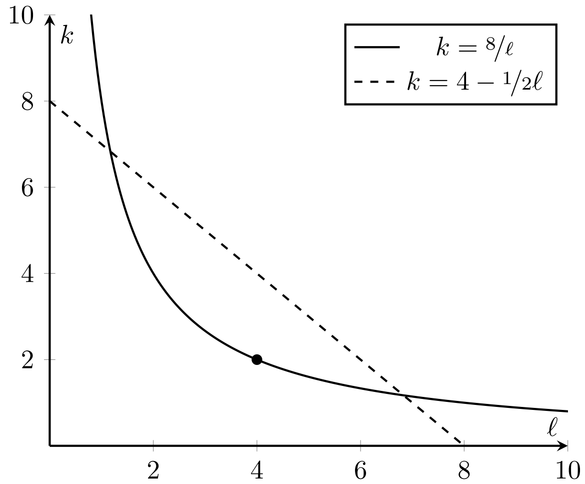
however, observe that as

$$r = 2 \to r = 1$$

cost function becomes

$$\begin{aligned}
k &= \frac{8}{r} - \frac{w}{r}\ell \\
k &= \frac{8}{1} - \frac{1}{1}\ell \\
k &= 8 - \ell
\end{aligned}$$

which graphically demonstrates



that the previous bundle $(4, 2)$ does not satisfy an optimality condition

$$\begin{cases} TRS(\ell, k) & = -{}^{w}/_{r} \\ f(\ell, k) & = x \end{cases}$$

anymore.

Question 1

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1.c
What is the new least-cost combination of inputs if r decreases to \$1?

Answer to 1.c

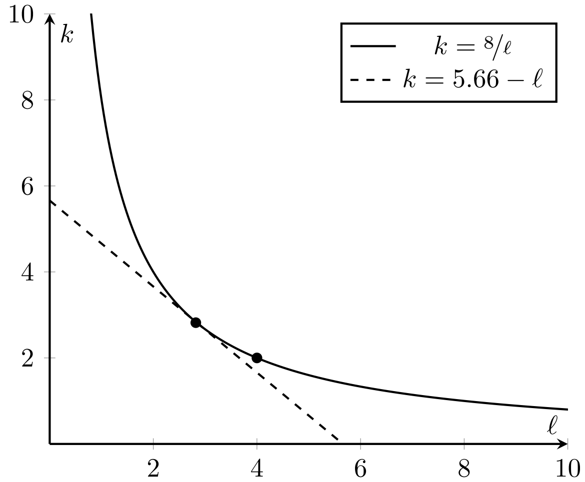
The new optimal bundle

$$\begin{cases} TRS(\ell, k) & = -{}^{w}/_{r} \\ f(\ell, k) & = x \end{cases} \Rightarrow \begin{cases} -{}^{k}/_{\ell} & = -{}^{1}/_{1} \\ \ell k & = 8 \end{cases}$$
$$\Rightarrow \begin{cases} \ell & = k \\ \ell k & = 8 \end{cases}$$
$$\Rightarrow \begin{cases} \ell & = k \\ kk & = 8 \end{cases}$$
$$\Rightarrow \begin{cases} \ell & = \sqrt{8} \\ k & = \sqrt{8} \end{cases}$$

to illustrate it graphically we need to calculate the new isocost

$$
\begin{aligned}
w\ell + rk &= 2\sqrt{8} \approx 5.66 \\
k &= \frac{5.66}{r} - \frac{w}{r}\ell \\
k &= 5.66 - \ell
\end{aligned}
$$

and combine it with an isoquant



Question 1

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1.d
How the production cost changed as r decreases to \$1?

Answer to 1.d

$$\begin{array}{rl} & w^{\text{old}}\ell^{\text{old}} + r^{\text{old}}k^{\text{old}} = 8 \\ - & \underline{w^{\text{new}}\ell^{\text{new}} + r^{\text{new}}k^{\text{new}} = 5.66} \\ & = 2.34 \end{array}$$

Question 2 (Profit Maximization in the Long Run)

A producer has to pay a licence fee of $1 to run his business. If the producer decides not to pay the fee, he essentially exits the market, in which case his cost of production is equal to zero. If he decides to pay the fee, then his cost of production will include both the licence fee and a component that depends on the level of output as well as on the the prices of input. In particular, let's assume that the (long run) cost function of this producer reads:

$$C(x,w,r) = \begin{cases} 0 & \text{if } x = 0 \\ 1 + \frac{1}{2}wrx^2 & \text{if } x > 0 \end{cases}$$

NOTE:
In the long run, a licence fee is an example of an economic fixed cost. It is an economic cost because in the long run the producer can decide whether to pay the fee or not (hence it is not a sunk cost). It is a fixed cost because it does not depend on the level of output. Clearly, once the fee has been paid, it becomes a fixed sunk cost and hence ceases to be an economic cost.

2.a

What is the profit maximizing quantity if

$$w = \$2$$
$$r = \$4$$
$$p = \$2?$$

Answer to 2.a

The necessary condition for profit maximization is

$$p = MC(x)$$

a point where the firm can not increase the profit by changing the level of output.[2]

[2] To see it assume that

$$p > MC(x),$$

i.e. firm is producing less. Recall that by definition $MC(y) = \frac{\partial C(x)}{\partial x} \approx \frac{\Delta C(x)}{\Delta x}$. Thus, the above can be rearranged into

$$p - \frac{\Delta C(x)}{\Delta x} > 0$$

now an output increase by Δy gives a profit increase of

$$p\Delta x - \frac{\Delta C(x)}{\cancel{\Delta x}}\cancel{\Delta x} > 0 \Rightarrow p\Delta x - \Delta C(x) > 0 \Rightarrow p\Delta x > \Delta C(x)$$

so it makes sense to increase the output, or, put differently, increase in revenue exceeds increase in costs. So we proved that $p > MC(x)$ can not be optimal. The same reasoning is true for $p < MC(x)$

So with

$$MC(x) = \frac{\partial C(x)}{\partial x} = \frac{\partial\left(1 + \frac{1}{2}wrx^2\right)}{\partial x} = \frac{\partial(1 + 4x^2)}{\partial x} = 8x \qquad x \neq 0$$

and

$$p = 2$$

we have

$$\begin{aligned} &p = MC(x) \\ &2 = 8x \\ &x = {}^1\!/_4 \end{aligned}$$

Note that the condition was necessary, i.e. all profit maximizing values satisfy $p = MC(x)$ but not other way around. We need to check the shutdown condition (a condition that all points need to satisfy, including the ones selected by profit maximizing condition)

$$p > AC(x)$$

as well[3]

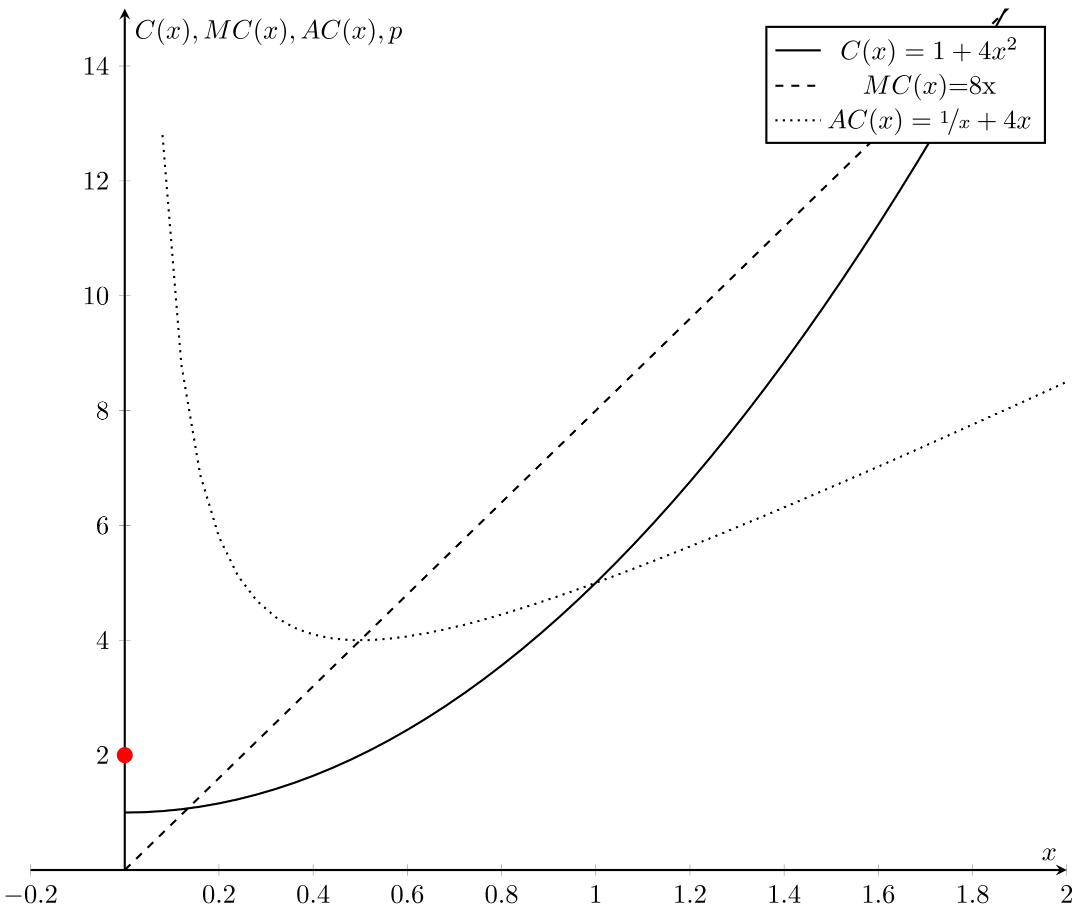
Clearly

$$AC(x) = \frac{C(x)}{x} = \frac{(1 + \frac{1}{2}wrx^2)}{x} = \frac{1 + 4x^2}{x} = \frac{1}{x} + 4x$$

and with $p = 2$ we have

$$\begin{aligned} &p > AC(x) \\ &2 > AC\left(\frac{1}{4}\right) \\ &2 \not> 5 \end{aligned}$$

The operation does not cover $AC(x)$, thus shutting down is optimal. Which is a red dot on the figure below



[3]You go out of business in long run if

$$\begin{aligned} \textit{Producing nothing} &> \textit{Producing something} \\ 0 &> \textit{Profits} \\ 0 &> px - C(x) \\ px &> C(x) \\ p &> AC(x) \end{aligned}$$

Question 2 (Profit Maximization in the Long Run)

A producer has to pay a licence fee of \$1 to run his business. If the producer decides not to pay the fee, he essentially exits the market, in which case his cost of production is equal to zero. If he decides to pay the fee, then his cost of production will include both the licence fee and a component that depends on the level of output as well as on the the prices of input. In particular, let's assume that the (long run) cost function of this producer reads:

$$C(x,w,r) = \begin{cases} 0 & \text{if } x = 0 \\ 1 + \frac{1}{2}wrx^2 & \text{if } x > 0 \end{cases}$$

NOTE:
In the long run, a licence fee is an example of an economic fixed cost. It is an economic cost because in the long run the producer can decide whether to pay the fee or not (hence it is not a sunk cost). It is a fixed cost because it does not depend on the level of output. Clearly, once the fee has been paid, it becomes a fixed sunk cost and hence ceases to be an economic cost.

2.b
What happens if the price of output increases to

$$p = \$8?$$

Answer to 2.b

Necessary

$$\begin{aligned} 8 &= MC(x) \\ 8 &= 8x \\ x &= 1 \end{aligned}$$

and sufficient

$$\begin{aligned} p &> AC(x) \\ 8 &> AC(1) \\ 8 &> 5 \end{aligned}$$

conditions are both satisfied

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2.c

The graph below shows the average cost curve of this producer. Find the break even price and complete the graph so that you eventually identify the supply curve of the producer.

Answer to 2.c

A break even point (a cost minimizing point)

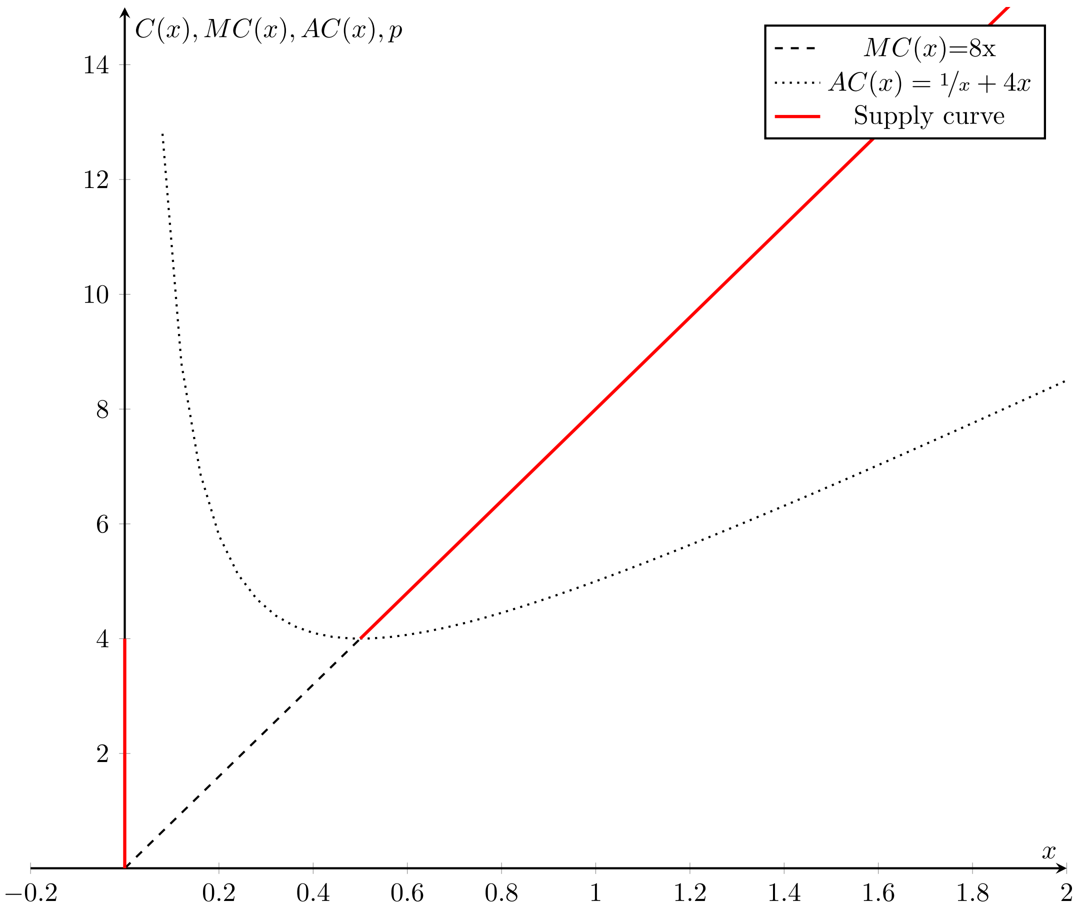
$$\begin{aligned} AC(x) &= MC(x) \\ {}^1\!/_x + 4x &= 8x \\ x &= {}^1\!/_2 \end{aligned}$$

Plugging into it into $AC(x)$ (or $MC(x)$) gives

$$AC({}^1\!/_2) = 2 + 2 = 4$$

to sum up

$$\begin{aligned} y^{be} &= {}^1\!/_2 \\ p^{be} &= 4 \end{aligned}$$



$C(x), MC(x), AC(x), p$
$MC(x)$=8x
$AC(x) = 1/x + 4x$
Supply curve
14
12
10
8
6
4
2
x
−0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Question 2 (Profit Maximization in the Long Run)

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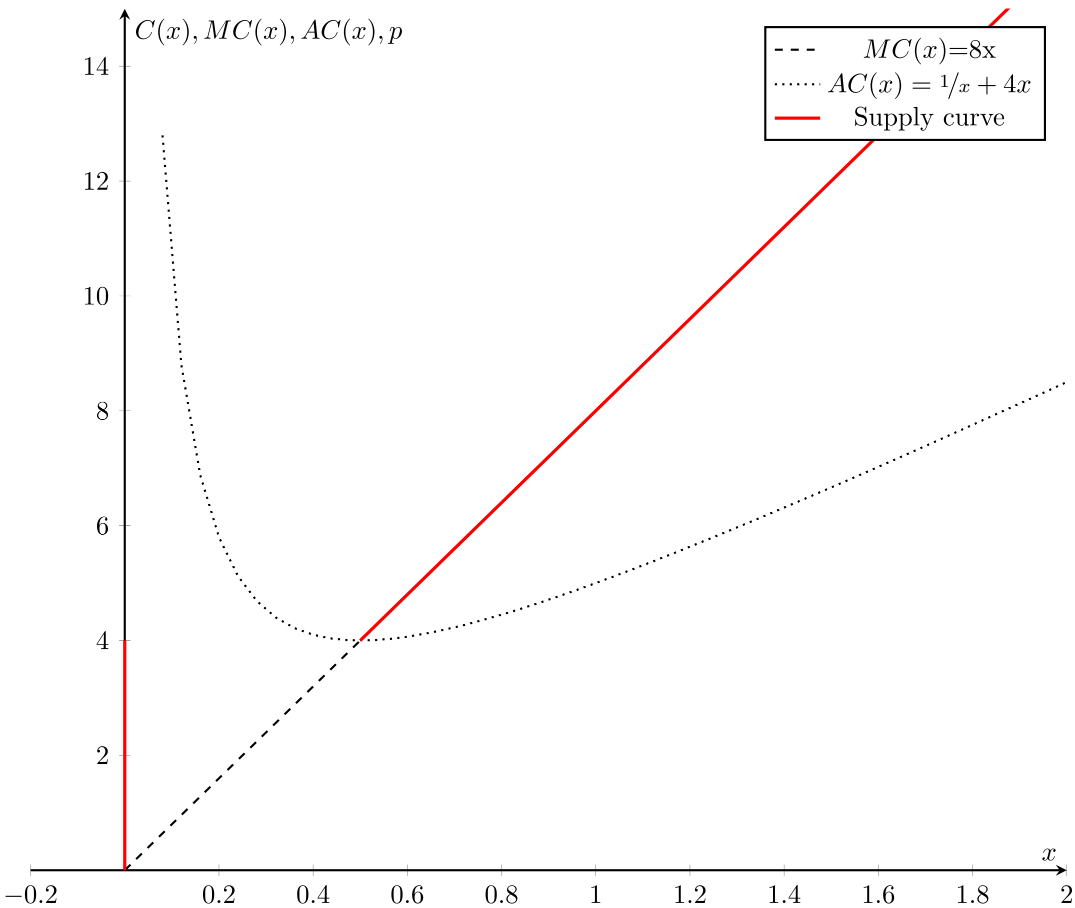
2.d
Write the mathematical expression of the supply curve

Answer to 2.d

$$S(p) = \begin{cases} 0 & \text{if } p < 4 \\ 8x & \text{if } p \geq 0 \end{cases}$$



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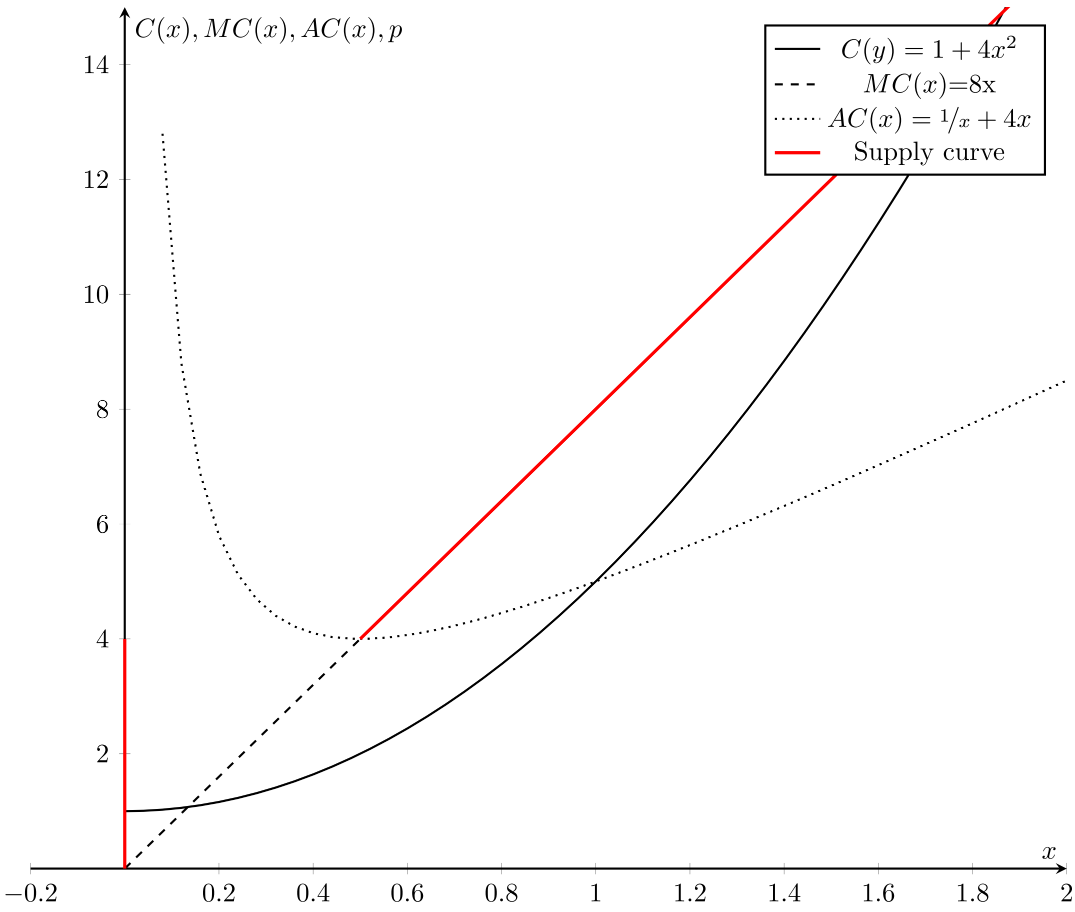
2.e.

What happens to the supply curve that you have represented graphically if w decreases? Explain.

Answer to 2.e



$C(x), MC(x), AC(x), p$
$C(y) = 1 + 4x^2$
$MC(x)$=8x
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Supply curve
14
12
10
8
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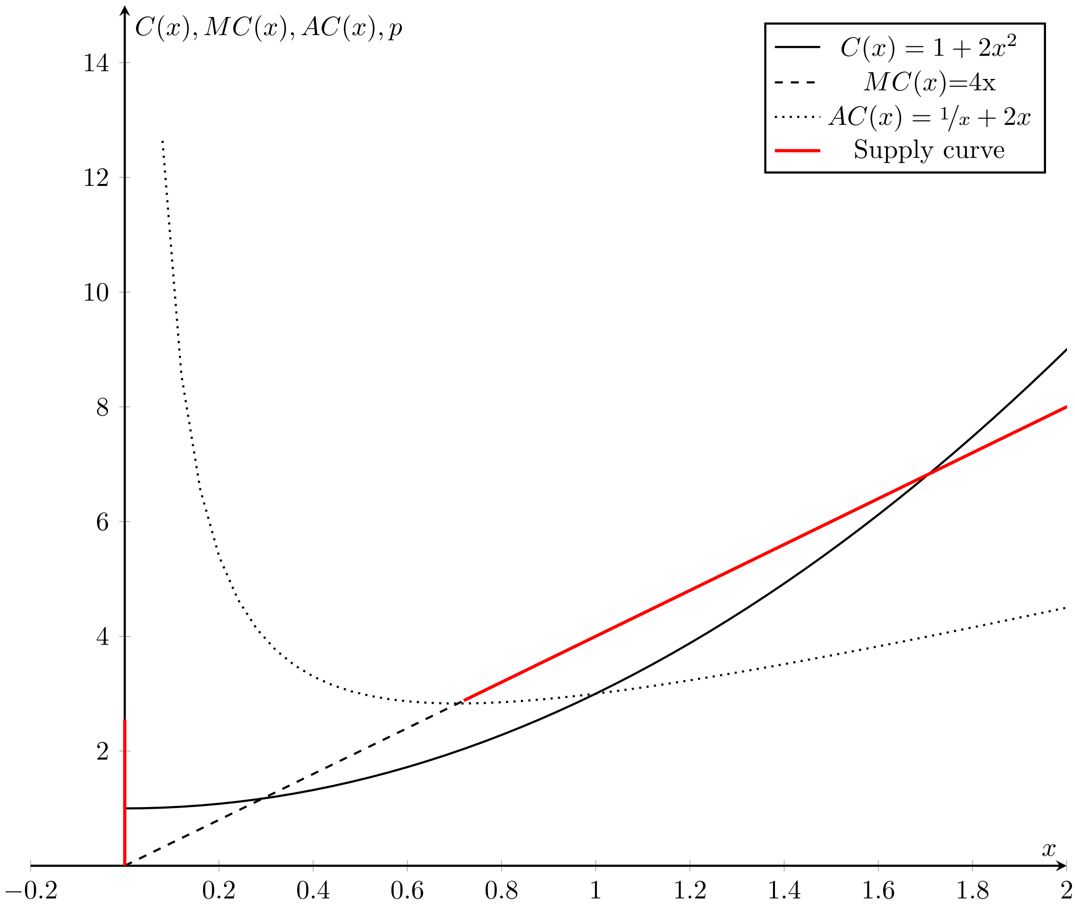
4
2
x
-0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



$C(x), MC(x), AC(x), p$
$C(x) = 1 + 2x^2$
$MC(x)$=4x
$AC(x) = {}^1/_x + 2x$
Supply curve
14
12
10
8
6
4
2
x
-0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Question 3 (Cost Minimization and Profit Maximization)

True or False? If a producer minimizes costs, she does not necessarily maximize profits; but if she maximizes profits, she also minimizes costs.

True. Cost minimization does not guarantee that the producer is choosing the specific level of output that maximizes profits. It only guarantees that if a given level of output is produced, it is produced at the lowest cost (in other words, the quantity being produced at the lowest cost may not be the quantity that also maximizes profits). On the other hand, if the producer is maximizing profits, he is necessarily minimizing costs. To see this assume that a given quantity x' is not produced at the lowest cost. Then for the producer there would be a way to produce x' at a lower cost and thus increase profits. Therefore x' cannot be the profit maximizing quantity.

References

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