Behavioral Economics

Eyster and Rabin (2010), "Naive Herding in Rich-Information Settings"

Sergey Alexeev

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University of Technology Sydney

1. There is a decision to be made – for example, whether to adopt a new technology, wear a new style of clothing, eat in a new restaurant, or support a particular political position;
2. People make the decision sequentially, and each person can observe the choices made by those who acted earlier;
3. Each person has some private information that helps guide their decision;
4. A person can't directly observe the private information that other people know, but he or she can make inferences about this private information from what they do.

Table of Content

Anderson and Holt (1996)

I: Classroom Games: Information Cascades

An urn that could be majority red or majority blue equally likely:

(a) The 1st student tells what what she sees. The guess conveys perfect information;
(b) If the 2nd student sees the same color – guess the same. If different – brakes the tie. The guess again conveys perfect information;
(c) If the 3rd Student sees opposite guess, then he guess what he sees. If the same then he saw three independent draws and ignores his information;
(d) If the 4th student (and onward) sees three identical guesses in a row and knows that the first 2 were true, while the 3rd is not informative, then she ignores her private information;

- Notes on: full rationality, potential non-optimality, fragility.

- Chances maximizing rule: $\text{Pr}[\text{mb}|\text{s\&h}] > \frac{1}{2}$
- Priors: $\text{Pr}[\text{mb}] = \text{Pr}[\text{mr}] = \frac{1}{2}$
- Conditionals: $\text{Pr}[\text{b}|\text{mb}] = \text{Pr}[\text{r}|\text{mr}] = \frac{2}{3}$
- The 1st student:

$$\begin{aligned}\text{Pr}[\text{mb}|\text{b}] &= \frac{\text{Pr}[\text{mb}]\cdot\text{Pr}[\text{b}|\text{mb}]}{\text{Pr}[\text{b}]} \\ &= \frac{\text{Pr}[\text{mb}]\cdot\text{Pr}[\text{b}|\text{mb}]}{\text{Pr}[\text{mb}]\cdot\text{Pr}[\text{b}|\text{mb}]+\text{Pr}[\text{mr}]\cdot\text{Pr}[\text{b}|\text{mr}]} = \frac{2}{3}\end{aligned}$$

- The 2nd student: ...
- The 3rd student:

$$\begin{aligned}\text{Pr}[\text{mb}|\text{b},\text{b},\text{r}] &= \frac{\text{Pr}[\text{mb}]\cdot\text{Pr}[\text{b,b,r}|\text{mb}]}{\text{Pr}[\text{b,b,r}]} \\ &= \frac{\text{Pr}[\text{mb}]\cdot\text{Pr}[\text{b,b,r}|\text{mb}]}{\text{Pr}[\text{mb}]\cdot\text{Pr}[\text{b,b,r}|\text{mb}]+\text{Pr}[\text{mr}]\cdot\text{Pr}[\text{b,b,r}|\text{mr}]} = \frac{2}{3}\end{aligned}$$

- Learning does not have to be Bayesian in the first place, but if it is:
$$B(h^*|e_1, e_2, ...) = \frac{\pi(e_1, e_2, ...|h^*)\pi(h^*)}{\sum_{h \in H} \pi(e_1, e_2, ...|h)\pi(h)}$$
it does not have to be perfect;

Eyster and Rabin (2010)

I: Naive Inference

- Best response trailing naive inference (BRTNI);
- Builds off of a weaker form of concept of "cursed equilibrium";
- Inferentially naive players infer "too much":
    - Inferential naivety push toward overweighting prior signals, its essential property, which drives the central results, is that herders end up placing far too much weight on early relative to late signals.
- The relative weight placed on different predecessors' signals *vs.* relative weight each person places on her own versus others' signals.

Eyster and Rabin (2010)

II: Rational and Naive Learning in a Rich Setting

- Rational herders either converge to only weak public beliefs or only very infrequently herd on the wrong action;
- Consider:
  - $\omega \in \{0, 1\}$;
  - $\Pr[\omega = 1] = \pi$;
  - I_t information (private and public) of t;
  - $Q_t \equiv E[\omega | I_t] = \Pr[\omega = 1 | I_t]$;
- (P1) bounds t's posterior belief $Q_t \geq q$ when $\omega = 0$:

$$\Pr[Q_t \geq q | \omega = 0] \leq \frac{\pi}{1-\pi} \frac{1-q}{q} \tag{P1}$$

- The maximum probability that t can hold information causing him to believe that $\omega = 1$ with at least probability q, when, in fact, $\omega = 0$;
- The bound holds in any binary-state Bayesian learning environment.

- In richer environment confident-yet-mistaken herd is even more limited;
- Consider:
  - $A = \left\{0, {}^{1}\!/_{n}, {}^{2}\!/_{n}, \ldots, {}^{(n-1)}\!/_{n}, 1\right\}$, $n+1$ set of actions
  - Assume $g_t(a;\omega) = -(a_t - \omega)^2$, with $\arg\max a_t = E[\omega|I_t]$
  - S is a denumerable set of signals;
  - $r \equiv \inf_{s\in S}\{\Pr[\omega = 1|s]\}$;
  - All predecessors' are observed and actions converge.
- Then with $\pi = 1/2$ the following holds:

$$\Pr[\lim_{x\to\infty} a_t = 1|\omega = 0] \leq \frac{r}{1-r}\frac{1}{2n-1} \tag{C2}$$

- With $n = 1$ cascade start only of public beliefs must exceed $1 - r$;
  - Negate to see;
- Then with (P1) a chance of mistaken herd cannot exceed ${}^{r}\!/_{(1-r)}$:
  - If $r = 0.05$ meaning only that once in a (very long) while some player receives a private signal strong enough to be 95 percent certain of the state being $\omega = 0$, then players can wrongly herd on $\omega = 1$ no more than $\simeq 5\%$ of the time.
- Finer action spaces reduce mistaken herding.

Proof of C2

To differentiate naive and rational learning consider the model:

- $\omega \in \{0, 1\}$, ex ante equally likely ;
- t in a countable infinite sequence receives $s_t \in [0, 1]$ which are *i.i.d* conditional on the state;
- Signal have continuous densities f_0 and f_1;
- Before taking action in $[0, 1]$, t observes signal and *all* previous actions;
- For simplicity: for each $s \in [0, 1]$, $f_0(s) = f_1(1 - s)$ and $L(s) \equiv f_1(s)/f_0(s)$ with image $\mathcal{R}_+$ and $L'(s) > 0$;
- Simplifications allow to normalize $s = \Pr[\omega = 1|s]$;
- Let $a_t(a_t, ..., a_{t-1}; s_t)$ be an action of t. Rich action space ensures that each player's action fully reveals her beliefs;
- Let $E[\omega|I_t] = \Pr[\omega = 1|I_t]$ a probabilistic belief of t with I_t that $\omega = 1$;
- Assume $g_t(a; \omega) = -(a_t - \omega)^2$, with $\arg\max a_t = E[\omega|I_t]$;
- t takes $a_t = 0$ if $E[\omega|I_t] = 0$ and $a_t = 1$ if $E[\omega|I_t] = 1$;

The analyzes of a rational player:

- P1 chooses $\ln(a_1/(1-a_1)) = \ln(s_1/(1-s_1))$
- P2 combines P1's action with his private information:

$$\ln\left(\frac{a_2}{1-a_2}\right) = \ln\left(\frac{a_1}{1-a_1}\right) + \ln\left(\frac{s_2}{1-s_2}\right) = \ln\left(\frac{s_1}{1-s_1}\right) + \ln\left(\frac{s_2}{1-s_2}\right)$$

- Interpretation: since agents share a common prior, P2 can adopt P1's posterior as his own prior before incorporating his private signal;
- That's why P3 does not benefit from observing P1 if P2 is seen;
- In general: $\ln(a_t/(1-a_t)) = \sum_{\tau \le t} \ln(s_\tau/(1-s_\tau))$
- Behaviorally: $\ln(a_t/(1-a_t)) = \ln(a_{t-1}/(1-a_{t-1})) + \ln(s_t/(1-s_t))$
- A note on t's unbounded likelihood ratio and continuum of actions.

BRTNI neglect their predecessors' inferences:

- P1 is not effected (no inference involved);
- P2 correctly infers P1's signal from her action (typical BNE):

$$\begin{aligned}\ln\left(\frac{a_2}{1-a_2}\right) &= \ln\left(\frac{a_1}{1-a_1}\right)+\ln\left(\frac{s_2}{1-s_2}\right)\\ &= \ln\left(\frac{s_1}{1-s_1}\right)+\ln\left(\frac{s_2}{1-s_2}\right)\end{aligned}$$

- P3 neglects how P2 incorporates P1 signal into his action:

$$\begin{aligned}\ln\left(\frac{a_3}{1-a_3}\right) &= \ln\left(\frac{a_1}{1-a_1}\right)+\ln\left(\frac{a_2}{1-a_2}\right)+\ln\left(\frac{s_3}{1-s_3}\right)\\ &= \ln\left(\frac{s_1}{1-s_1}\right)+\left(\ln\left(\frac{s_1}{1-s_1}\right)+\ln\left(\frac{s_2}{1-s_2}\right)\right)+\ln\left(\frac{s_3}{1-s_3}\right)\\ &= 2\ln\left(\frac{s_1}{1-s_1}\right)+\ln\left(\frac{s_2}{1-s_2}\right)+\ln\left(\frac{s_3}{1-s_3}\right)\end{aligned}$$

- Generally:

$$\ln\left(\frac{a_t}{1-a_t}\right)=\left[\sum\nolimits_{\tau<t}2^{t-1-\tau}\ln\left(\frac{s_\tau}{1-s_\tau}\right)\right]+\ln\left(\frac{s_t}{1-s_t}\right)$$

- BRTNI play allows a failure of learning of true state even with unbounded signal strength and arbitrary large number of Ps:

  In BRTI play, for each $r < 1$, *there exist* $\delta > 0$, *such that:*
  $\Pr[a_t > r, \forall t | \omega = 0] > \delta$ (P3)

- Even when $\omega = 0$ it is possible that BRTNI in an infinite sequence chooses and action that exceeds any given threshold;
- If the first couple of agents receive signals high enough to take actions above r, then with positive probability no agent ever takes an action below r;
- Driven by the speed of forming a believe that $\omega = 0$ is a true state.

Proof of P3

- Unlike rational beliefs, BRTNI beliefs do not form a martingale:
  - When public belief $P_t > 1/2$, then $E[P_{t+1}|P_t] > P_t$
  - When public belief $P_t < 1/2$, then $E[P_{t+1}|P_t] < P_t$
- Beliefs drift in this predictable way because BRTNI players in future periods reweigh information already contained in current beliefs and become fully confident in the wrong state:

$$\textit{BRTI actions and beliefs converge almost surely to } 0 \textit{ or } 1 \tag{P4}$$

Proof of P4

Simulations with $f_o(s) = 2(1-s)$ and $f_1(s) = 2s$ (when $\omega = 1$)

	BNE			BRTNI		
Player	$a \leq 0.05$	$0.05 < a \leq 0.95$	$a > 0.95$	$a \leq 0.05$	$0.05 < a \leq 0.95$	$a > 0.95$
1	0.0026	0.8998	0.0976	0.0025	0.8998	0.0977
2	0.0060	0.6905	0.3035	0.0058	0.6912	0.3030
3	0.0070	0.5059	0.4871	0.0216	0.3819	0.5965
4	0.0069	0.3684	0.6247	0.0483	0.1877	0.7640
5	0.0060	0.2708	0.7232	0.0739	0.0929	0.8332
6	0.0051	0.1995	0.7954	0.0914	0.0463	0.8623
7	0.0041	0.1482	0.8477	0.1016	0.023	0.8754
8	0.0033	0.111	0.8857	0.1068	0.0117	0.8815
9	0.0026	0.0826	0.9148	0.1098	0.0057	0.8845
10	0.0020	0.0624	0.9356	0.1115	0.0029	0.8856

notes:

- likelihood that both types P2 take a low action is $\simeq 0.006$;
- RP3 likely than not chooses a higher action than RP2 since when $\omega = 1$ most signals move posteriors in that direction. Indeed, for RP2 and RP3 take low actions is similar;
- NP3, however, three times as likely as their predecessors to choose a low action;
- intuitively, because they interpret NP1 and NP2 low actions as two strong and independent pieces of evidence in favor of $\omega = 0$;
- only very high signals can swing actions above 0.05;
- when $\omega = 1$ NPs converge to $a = 0$ with $\simeq 11\%$, while it occur with RPs only $\simeq 2\%$;
- there is 99.7% chance of NP10 taking $a \leq 0.05$ and $a > 0.95$ (against 93.8%).

- BRTNI play converge on the wrong limiting action with positive probability ((P3) and (P4));
  - On contrary rational players almost surely converge on the right action;
- Another interesting feature is that rational players always benefit on average from observing, while BRTNI may not;
  - If expected cost of overconfidence exceeds the added information in others' actions;
- $g_k(a_k, \omega) = -(a - \omega)^{2n}$, higher n more costly it is to chose action distant from the true state;
- Belief are n invariant and wrong limiting action is reached 11 $(= 1/9)$ % of time;
- A long-run average payoff is $-(1)^{2n} = -1 \times 1/9 = -1/9$;
- A lower bound on average payoff is $-(1/2)^{2n}$, so if $n = 1$ learning is good, while for $n \geq 2$ not so much, since $-(1/2)^{2n} \geq -1/9$.

Eyster and Rabin (2010)

III: Harmful Learning with Long-Run Agents

- People may choose actions repeatedly;
- Consider:
  - Player $\{A, B, C\}$ move in sequence $A, B, C, A, B, C, A \ldots$;
  - Each player's growing collection of private signals almost surely reveals the state;
  - Rational and naive would choose the right action if acted solely, yet...

Suppose that three long-run BRTNI players $\{A, B, C\}$ move in sequence $A, B, C, A \ldots$ Then for each $r \in (0,1)$ there exist $\delta > 0$ such that

$$\Pr\left[\left(\frac{a_t}{1-a_t}\right) > e^t \left(\frac{r}{1-r}\right), \forall t | \omega = 0\right] > \delta \tag{P6}$$

- When $\omega = 0$, for $r > 1/2$, it happens that all long-run BRTNI players play actions above r and converge to certain beliefs that $\omega = 1$.

Proof of P6

Hurray! We are done!

Banerjee (1992)

I: A Simple, General Cascade Model

- A simple, cascade model consists of:
  1. States of the world: $\Pr[G] = p$ and $\Pr[B] = 1 - p$;
  2. Payoffs: reject $\rightarrow 0$ or accept $\rightarrow v_g p + v_b(1-p)(=0)$
  3. Signals: $q > \frac{1}{2}$ (e.g. more reviews for a better restaurant)

$$\Pr[H|G] = q \quad \Leftrightarrow \quad \Pr[L|G] = 1 - q$$
$$\Pr[L|B] = q \quad \Leftrightarrow \quad \Pr[H|B] = 1 - q$$

States

Signals		B	G
	L	q	$1-q$
	H	$1-q$	q

back to general model

- Individual decision:
  - A high signal shifts expected payoff:
    $v_g \Pr[G] + v_b \Pr[B] = 0 \rightarrow v_g \Pr[G|H] + v_b \Pr[B|H]$

$$\begin{aligned} \Pr[G|H] &= \frac{\Pr[G]\cdot\Pr[H|G]}{\Pr[H]} \\ &= \frac{\Pr[G]\cdot\Pr[H|G]}{\Pr[G]\cdot\Pr[H|G]+\Pr[B]\cdot\Pr[H|B]} \\ &= \frac{pq}{pq+(1-p)(1-q)} > p^* \end{aligned}$$

- Multiple agents:
  - Define S as a set of signals with a high and b low signals then:

$$\begin{aligned} \Pr[G|S] &= \frac{\Pr[G]\cdot\Pr[S|G]}{\Pr[S]} \\ &= \frac{pq^a(1-q)^b}{pq^a(1-q)^b+(1-p)(1-q)^a q^b}^\dagger \end{aligned}$$

  - Implying:

$$\begin{aligned} a > (<) b &\Rightarrow \Pr[G|S] > (<) \Pr[G] \\ a = b &\Rightarrow \Pr[G|S] = \Pr[G] \end{aligned}$$

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*Note that $pq + (1-p) < pq + (1-p)q = q$

†Replace second term in denominator with $(1-p)q^a(1-q)^b$

Banerjee (1992)

II: Sequential Decision-Making and Cascades

- Recall that if P1 and P2 made opposite decisions P3 follows his signal. And future Ps know that;
- If P1 and P2 made the same decision then all do the same;
- If number of acceptance differ from number of rejections by at most one, person follows the signal;
- But once the difference is bigger, everyone follows the majority;
- The difference won't stay within $(-1, 1)$ for long:
    - Divide N into three consecutive players;
    - People in a block receive the same signal with probability: $q^3 + (1-q)^3$
    - The probability that none of these blocks consist of the same signal: $(1 - q^3 - (1-q)^3)^{N/3}$
    - And goes to 0 as $N \to \infty$

References

- Anderson, L. R. and C. A. Holt (1996). "Classroom Games: Information Cascades". In: *The Journal of Economic Perspectives* 10.4, pp. 187–193. ISSN: 08953309. URL: http://www.jstor.org/stable/2138561.
- Banerjee, A. V. (1992). "A Simple Model of Herd Behavior". In: *The Quarterly Journal of Economics* 107.3, pp. 797–817. ISSN: 00335533, 15314650. URL: http://www.jstor.org/stable/2118364.
- Eyster, E. and M. Rabin (2010). "Naive Herding in Rich-Information Settings". In: *American Economic Journal: Microeconomics* 2.4, pp. 221–243. ISSN: 19457669, 19457685. URL: http://www.jstor.org/stable/25760414.

Technical appendix

The Proof of Proposition 1

Let $\bar{I}_t = \{I_t = (s_t; a_1, ..., a_{t-1}) : Q_t \geq q\}$. From Bayes' Rule,

$$\begin{aligned}
\Pr[\omega = 1|\bar{I}_t] &= \frac{\pi}{\pi + (1-\pi)\frac{\Pr[\bar{I}_t|\omega=0]}{\Pr[\bar{I}_t|\omega=1]}} \geq q \\
\Rightarrow \frac{\Pr[\bar{I}_t|\omega=0]}{\Pr[\bar{I}_t|\omega=1]} &\leq \frac{\pi}{1-\pi}\frac{1-q}{1}
\end{aligned}$$

Because $\Pr[\bar{I}_t|\omega = 1] \leq 1$, $\Pr[\bar{I}_t|\omega = 0] \leq \frac{\pi}{1-\pi}\frac{1-q}{1}$

back to P1

The Proof of Corollary 2

When public beliefs are that $\Pr[\omega = 1|(a_t, ..., a_{t-1})] = p$, player t with private belief r takes action $a_t = 1$ iff:

$$\begin{aligned} \Pr[\omega = 1|I_t] = \tfrac{pr}{pr+(1-p)(1-r)} &\geq \tfrac{2n-1}{2n} \\ p &\geq \frac{1}{1+\frac{r}{1-r}\frac{1}{2n-1}} \end{aligned}$$

Then (P1) with $q = \frac{1}{1+\frac{r}{1-r}\frac{1}{2n-1}}$ and $\pi = \frac{1}{2}$ gives (C2)

back to C2

Bayesian Updating as a Likelihood Ration (Bayes Factor)

With binary sample space the odds of E are: $O(E) = \frac{P(E)}{P(E^c)}$

- Think of a flip of a fair coin;
- $P(E) = p \Rightarrow O(E) = {}^{p}\!/_{1-p}$

Bayesian updating – in the language of odds – is prior odds updated to posterior odds:

$$
\begin{aligned}
\text{Bayes factor} = O(H|D) &= \frac{P(D|H)}{P(D|H^c)} \\
&= \frac{P(D|H) \cdot P(H)}{P(D|H^c) \cdot P(H^c)} \\
&= \frac{P(D|H)}{P(D|H^c)} \cdot \frac{P(H)}{P(H^c)} \\
&= \frac{P(D|H)}{P(D|H^c)} \cdot O(H) \\
\text{posterior odds} &= \text{Bayes factor} \times \text{prior odds}
\end{aligned}
$$

Log odds are more convenient in practice:

$$
\begin{aligned}
O(H|D_1, D_2) &= BF_2 \cdot BF_1 \cdot O(H) \\
\ln(O(H|D_1, D_2)) &= \ln(BF_2) + \ln(BF_1) + \ln(O(H))
\end{aligned}
$$

back to naivety

The Proof of Proposition 3 (beginning)

Pick $r \in (1/2, 1)$, define $R = \ln(1/(1-r)) > 0$, let P_t be a log likelihood of public belief at period t.

With BRTNI play $P_{t+1} = 2P_t + \ln(s_t/(1-s_t))$

When $\omega = 0$, with positive probability $P_2 \geq 3R$

If $\ln(s_t/(1-s_t)) > -tR \; \forall t$ then:

$$\begin{aligned} P_3 = 2P_2 + \ln(s_2/(1-s_2)) &> 2 \times 3R - 2R = 4R \text{ and} \\ P_4 = 2P_3 + \ln(s_3/(1-s_3)) &> 2 \times 4R - 3R = 5R, \text{ etc.} \end{aligned}$$

In general:

$$\begin{aligned} P_t &> (t+1)R \text{ and so} \\ \ln(a_t/(1-a_t)) = P_t + \ln(s_t/(1-s_t)) &> (t+1)R - tR = R \end{aligned}$$

Now...

back to P3

The Proof of Proposition 3 (continuation)

$$\begin{aligned} \Pr\left[\ln\left(S_t/(1-S_t)\right) < -tR | \omega = 0\right] &< \Pr\left[\left|\ln\left(S_t/(1-S_t)\right)\right| > tR \,|\omega = 0\right] \\ &\overset{\ddagger}{<} 1/(tR)^2 E\left[\left|\ln\left(S_t/(1-S_t)\right)\right|^2 |\omega = 0\right] \end{aligned}$$

Also,

$$\begin{aligned} Q \equiv E\left[\left(\ln\left(S/(1-S)\right)\right)^2 |\omega = 0\right] &= \int_0^1 \left(\ln\left(s/(1-s)\right)\right)^2 f_0(s) ds \\ &\leq M \int_0^1 \left(\ln\left(s/(1-s)\right)\right)^2 ds \\ &= M\left(\pi^2/3\right) \end{aligned}$$

for $M \equiv \sup\{f_0(s) : s \in [0,1]\}$, which is finite by the continuity of f_0

‡ Markov inequality: $\Pr[X \geq a] \leq E[X]/a$ if X is nonnegative r.v and $a > 0$

back to P3

The Proof of Proposition 3 (finale)

Define $\tau = \min\{t \in \mathcal{N} : Q < t^2R^2\}$ so that for each $t \geq \tau$, $((t^2R^2 - Q)/t^2R^2) \in (0,1)$, and let $C(R) \equiv \prod_{t=1}^{\tau-1}(1 - F_0(-tR)) > 0$.

$$\begin{aligned}
\Pr\left[{}^{S_t}\!/_{1-S_t} > e^{-tR}, \forall t \,|\omega = 0\right] &> C(R)\prod_{t\geq\tau} {}^{(t^2R^2-Q)}\!/_{(t^2R^2)} \\
&= C(R)\exp\left\{\sum_{t\geq\tau} {}^{(t^2R^2-Q)}\!/_{(t^2R^2)}\right\} \\
&= C(R)\exp\left\{\sum_{t\geq\tau} -{}^{Q}\!/_{z_t}\right\}
\end{aligned}$$

for $z_t \in (t^2R^2 - Q, t^2R^2)$, by the Mean-Value Theorem. Then,

$$\begin{aligned}
\Pr\left[{}^{S_t}\!/_{1-S_t} > e^{-tR}, \forall t \,|\omega = 0\right] &> C(R)\exp\left\{\sum_{t\geq\tau} -{}^{Q}\!/_{t^2R^2}\right\} \\
&> C(R)\exp\left\{\sum_{t\geq 1} -{}^{Q}\!/_{t^2R^2}\right\} \\
&= C(R)\exp\{-({}^{Q\pi}\!/_{6R^2})\} > 0
\end{aligned}$$

back to P3

The Proof of Proposition 4

From above, write: $2^{1-t}P_t = \sum_{\tau<t} 2^{-\tau} \ln\left(s_\tau/1-s_\tau\right)$

Since the three series

$$\begin{aligned}
\sum_{\tau=1}^{\infty} E\left[2^{-\tau}\ln\left(S/1-S\right)|\omega=0\right] &=^{\S} 2E\left[\ln\left(S/1-S\right)|\omega=0\right] \\
\sum_{\tau=1}^{\infty} \mathrm{var}\left[2^{-\tau}\ln\left(S/1-S\right)|\omega=0\right] &=^{\P} 1/3\,\mathrm{var}\left[\ln\left(S/1-S\right)|\omega=0\right] \\
\sum_{\tau=1}^{\infty} \mathrm{var}\left[2^{-\tau}\ln\left(S/1-S\right)|\geq 1\right] &=^{\|} \sum_{\tau=1}^{\infty} 4^{-\tau}\mathrm{var}\left[\ln\left(S/1-S\right)|\omega=0\right]
\end{aligned}$$

Kolmogorov's Three-Series Theorem implies that $2^{1-t}P_t$ converges a.s.

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§ Follow from finiteness of the second moment (and therefore the first)

¶ See above

‖ by Chebyshev's inequality

back to P4

The Proof of Proposition 6

back to P6

Some Simple Algebra

$$\begin{aligned} \ln\left(\frac{a_t}{1-a_t}\right) &\equiv A_t \\ \ln\left(\frac{s_t}{1-s_t}\right) &\equiv S_t \end{aligned}$$

Rational:

$$\begin{aligned} A_3 &= A_1 + S_2 + S_3 \\ \because A_3 &= A_2 + S_3 \\ A_2 &= A_1 + S_2 \\ A_1 &= S_1 \end{aligned}$$

Naive:

$$\begin{aligned} A_3 &= A_1 + A_2 + S_3 \\ &= S_1 + [S_1 + S_2] + S_3 \\ \because A_2 &= S_1 + S_2 \end{aligned}$$

Rational player give all signals equal weight, BRTNI overweight early signals, giving the first signal half of weight of all signals, the second half of what remain etc.

$$A_3 = S_1 + S_2 + S_3$$

$$A_3 = S_1 + S_1 + S_2 + S_3$$

back to naive inference